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Target Mass Effects in QCD Bjorken Sum Rule

Hiroyuki KAWAMURA and Tsuneo UEMATSU*

Department of Fundamental Sciences
FIHS, Kyoto University
Kyoto 606-01, JAPAN

Abstract

We investigate the target mass effects in QCD Bjorken sum rule. The magnitude of the target mass correction is estimated in a variety of methods employing positivity bound as well as the experimental data for the asymmetry parameters. It turns out that the target mass correction is sizable at low Q^2 of the order of a few GeV^2, where the QCD correction is significant. We show that there exists uncertainty due to target mass effects in determining the QCD effective coupling constant $\alpha_s(Q^2)$ from the Bjorken sum rule.

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There has been a lot of interest in the nucleon spin structure functions which can be measured by the deep inelastic scattering of polarized leptons on polarized nucleon targets [1, 2]. The nucleon spin structure is described by the two spin structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$. Recent experiments on the $g_1(x, Q^2)$ for the deuteron, ^{3}He and proton targets at CERN and SLAC [3, 4, 5] together with EMC data [2] have provided us with the data for testing the Bjorken sum rule [6] as well as the Ellis-Jaffe sum rule [7], and also for studying Q^2 evolution of $g_1(x, Q^2)$ [8].

In the framework of the operator product expansion and the renormalization group, the Bjorken sum rule with QCD radiative corrections reads:

$$\int_0^1 dx[g_1^p(x, Q^2) - g_1^n(x, Q^2)] = \frac{1}{6}\frac{G_A}{G_V}[1 - \frac{\alpha_s(Q^2)}{\pi} + \mathcal{O}(\alpha_s^2)]. \tag{1}$$

where $g_1^p(x, Q^2)$ and $g_1^n(x, Q^2)$ are the spin structure function of proton and neutron, respectively, with x and Q^2 being the Bjorken variable and the virtual photon mass squared. On the right-hand side, $G_A/G_V \equiv g_A$ is the ratio of the axial-vector to vector coupling constants. The first order QCD correction was calculuated in [9, 10, 11] and the higher order corrections were given in [12, 13, 14, 15].

In order to confront the QCD prediction with the experimental data at low Q^2 where the QCD corrections are significant, we have to take into account the corrections due to the mass of the target. In this Q^2 region where we cannot neglect the order M^2/Q^2 terms, with M being the nucleon mass, we have to extract the definite spin contribution in the operator product expansion. This can be achieved by considering the Nachtmann moments of the structure functions [16].

Some years ago, the target mass effects for polarized deep inelastic scattering were studied in refs. [17, 18]. The Nachtmann moments for the twist two and three operators in operator product expansion relevant to polarized deep inelastic scattering were obtained in closed analytic forms in ref. [18]. Taking the first Nachtmann moment we can extract the contribution from the spin 1 and twist-2 operator, and the Bjorken

sum rule with target mass corrections reads [18]:

$$\frac{1}{9}\int_0^1 dx\frac{\xi^2}{x^2}\Big[5+4\sqrt{1+\frac{4M^2x^2}{Q^2}}\Big]\Big[g_1^p(x,Q^2)-g_1^n(x,Q^2)\Big]$$
$$-\frac{4}{3}\int_0^1 dx\frac{\xi^2}{x^2}\frac{M^2x^2}{Q^2}\Big[g_2^p(x,Q^2)-g_2^n(x,Q^2)\Big]=\frac{1}{6}\frac{G_A}{G_V}\Big[1-\frac{\alpha_s(Q^2)}{\pi}+O(\alpha_s^2)\Big], \quad (2)$$

where M denotes the nucleon mass and the variable ξ [19] is given by

$$\xi=\frac{2x}{1+\sqrt{1+4M^2x^2/Q^2}}. \quad (3)$$

Note that in the presence of target mass correction, the other spin structure function $g_2^{p,n}(x,Q^2)$ also comes into play in the Bjorken sum rule. It should also be noted that target mass corrections considered as the expansion in powers of M^2/Q^2 is not valid when M^2/Q^2 is of order unity [20, 21].

Taking the difference between the left-hand side of (2) and that of (1), we get the target mass correction $\Delta\Gamma$ due to the necessary projection onto the definite spin as

$$\begin{aligned}\Delta\Gamma &= \int_0^1 dx\{\frac{5}{9}\frac{\xi^2}{x^2}+\frac{4}{9}\frac{\xi^2}{x^2}\sqrt{1+\frac{4M^2x^2}{Q^2}}-1\}\times\Big[g_1^p(x,Q^2)-g_1^n(x,Q^2)\Big]\\ &\quad -\frac{4}{3}\int_0^1 dx\frac{\xi^2}{x^2}\frac{M^2x^2}{Q^2}\Big[g_2^p(x,Q^2)-g_2^n(x,Q^2)\Big]. \quad (4)\end{aligned}$$

We now study the size of the target mass correction $\Delta\Gamma$ to the Bjorken sum rule. First we note that in terms of virtual photon asymmetry parameters A_1 and A_2, the structure functions are given by

$$\begin{aligned}g_1(x,Q^2) &= \frac{F_2(x,Q^2)[A_1(x,Q^2)+\gamma A_2(x,Q^2)]}{2x[1+R(x,Q^2)]},\\ g_2(x,Q^2) &= \frac{F_2(x,Q^2)[-A_1(x,Q^2)+A_2(x,Q^2)/\gamma]}{2x[1+R(x,Q^2)]}, \quad (5)\end{aligned}$$

where $F_2(x,Q^2)$ is the unpolarized structure function, $\gamma=\sqrt{4M^2x^2/Q^2}$ and $R(x,Q^2)=\sigma_L(x,Q^2)/\sigma_T(x,Q^2)$ is the ratio of the longitudinal to transverse virtual photon cross sections.

The target mass correction turns out to be

$$\begin{aligned}\Delta\Gamma &= \int_0^1 dx\Big\{\frac{5}{9}\frac{\xi^2}{x^2}+\frac{4}{9}\frac{\xi^2}{x^2}\sqrt{1+4M^2x^2/Q^2}-1\Big\}\\ &\quad\times\frac{1}{2x}\Big\{\frac{F_2^p(A_1^p+\gamma A_2^p)}{1+R^p}-\frac{F_2^n(A_1^n+\gamma A_2^n)}{1+R^n}\Big\}\\ &\quad-\frac{4}{3}\int_0^1 dx\frac{\xi^2}{x^2}\frac{M^2x^2}{Q^2}\frac{1}{2x}\Big\{\frac{F_2^p(-A_1^p+A_2^p/\gamma)}{1+R^p}-\frac{F_2^n(-A_1^n+A_2^n/\gamma)}{1+R^n}\Big\}. \qquad (6)\end{aligned}$$

We estimate the target mass correction $\Delta\Gamma$ in a variety of methods. First of all, we apply the positivity bound for the asymmetry parameters [22]:

$$|A_1|\le 1, \qquad |A_2|\le\sqrt{R}, \qquad (7)$$

to get the upper bound for $\Delta\Gamma$ (Analysis I).

In this case we have

$$\begin{aligned}|\Delta\Gamma| &\le \int_0^1 dx\frac{1}{2x}\Big|\frac{5}{9}\frac{\xi^2}{x^2}+\frac{4}{9}\frac{\xi^2}{x^2}\sqrt{1+\frac{4M^2x^2}{Q^2}}+\frac{4}{3}\frac{\xi^2}{x^2}\frac{M^2x^2}{Q^2}-1\Big|\times\Big(\frac{F_2^p}{1+R^p}+\frac{F_2^n}{1+R^n}\Big)\\ &+ \int_0^1 dx\frac{1}{2x}\Big|\Big(\frac{5}{9}\frac{\xi^2}{x^2}+\frac{4}{9}\frac{\xi^2}{x^2}\sqrt{1+\frac{4M^2x^2}{Q^2}}-1\Big)\sqrt{4M^2x^2/Q^2}\\ &\quad-\frac{4}{3}\frac{\xi^2}{x^2}\frac{M^2x^2}{Q^2}\frac{1}{\sqrt{4M^2x^2/Q^2}}\Big|\times\Big(\frac{F_2^p\sqrt{R^p}}{1+R^p}+\frac{F_2^n\sqrt{R^n}}{1+R^n}\Big). \qquad (8)\end{aligned}$$

Using the parametrization for R taken from the global fit of the SLAC data [23] and the NMC parametrization for $F_2(x,Q^2)$ [24], we obtain for the average Q^2 of the E142 data, $Q^2=2.0$ GeV^2 and the SMC data, $Q^2=4.6$ GeV^2;

$$|\Delta\Gamma|\le 0.036 \quad \text{for } Q^2=2.0 \text{ GeV}^2, \qquad |\Delta\Gamma|\le 0.0177 \quad \text{for } Q^2=4.6 \text{ GeV}^2, \qquad (9)$$

whereas the value of the SMC experiment is $\Gamma\equiv\Gamma_1^p-\Gamma_1^n=0.20\pm0.05\pm0.04$(syst.), where $\Gamma_1^{p(n)}=\int_0^1 dx g_1^{p(n)}(x,Q^2)$ [25]. In (9), the error of the upper bounds of $\Delta\Gamma$ due to the parametrizations R and F_2 is expected to be around 10 %.

In Fig.1 we have plotted the upper bound for $\Delta\Gamma$, which we denote by $\Delta\Gamma_{u.b.}$ (i.e. $|\Delta\Gamma|\le\Delta\Gamma_{u.b.}$), as a function of Q^2 for Analysis I by a solid line.

So far we have not made use of any experimental data on spin asymmetries A_1 and A_2. Now we employ the experimetal data to improve the upper bound. We take the data on A_1^p from SMC data [5] together with EMC data [2] and those for A_1^d from SMC group [3] to extract A_1^n, for which we can also use the E142 data [4]. As for the $A_2^{p,n}$ there are two possibilities : i) Either use the positivity bound $|A_2| \leq \sqrt{R}$ both for the proton and the neutron, or ii) use the recently measured A_2^p by the SMC group [26] together with the positivity bound for A_2^n.

For the choice i) which we call Analysis II, the upper bound for the $\Delta\Gamma$, $\Delta\Gamma_{u.b.}$, is shown in Fig.1 by the short-dashed line, which is located slightly lower than $\Delta\Gamma_{u.b.}$ for Analysis I. This situation can be understood by the following observation. If we decompose the $\Delta\Gamma_{u.b.}$ into two parts, $\Delta\Gamma_1$ and $\Delta\Gamma_2$, which are the contributions from A_1 and A_2, respectively, it turns out that $\Delta\Gamma_2$ is much larger than $\Delta\Gamma_1$. The value of $\Delta\Gamma_1$ turns out be less than 10 % of $\Delta\Gamma_2$. Furthermore we note that the integrals are not so sensitive to the parametrization of A_1, if we assume the Regge behavior for $x \sim 0$, and $A_1 \to 1$ for $x \to 1$. Here we have also assumed that there is no significant Q^2 dependence in A_1 as observed in the experiments [1-5].

For the choice ii) which will be called Analysis III, we have also plotted the upper bound in Fig.1 by the long-dashed line. Here we took the data on A_2^p obtained by SMC group at the first measurement of transverse asymmetries [26], where the number of data points are still four and the relative error bars are not so small. The A_2^p measured is much smaller than the positivity bound. If the A_2 for the neutron is also small as mentioned in ref.[4], the $\Delta\Gamma_{u.b.}$ becomes very small.

Now we turn to the issue related to the determination of the QCD coupling constant from Bjorken sum rule which has recently been discussed by Ellis and Karliner [27]. Up to the $\mathcal{O}(\alpha_s^4)$ we have the following QCD corrections [13, 14, 15]:

$$\Gamma(Q^2) = \frac{1}{6}\frac{G_A}{G_V}\Big[1-\frac{\alpha_s(Q^2)}{\pi}-3.5833(\frac{\alpha_s(Q^2)}{\pi})^2 -20.2153(\frac{\alpha_s(Q^2)}{\pi})^3-\mathcal{O}(130)(\frac{\alpha_s(Q^2)}{\pi})^4+\cdots\Big]. \quad (10)$$

By putting $\frac{1}{6}G_A/G_V = 0.2095$ and taking the left-hand side of (10) at $Q^2 = 2.5\text{GeV}^2$ to be 0.161, which was obtained by Ellis and Karliner in their analysis of E142 and E143 data [27], we find $\alpha_s(Q^2 = 2.5\text{GeV}^2) = 0.375$ [27]. The $Q^2 = 2.5\text{GeV}^2$ is the averaged value of the mean Q^2 of the E142 data ($< Q^2 > \simeq 2\text{GeV}^2$) and the preliminary E143 data ($< Q^2 > \simeq 3\text{GeV}^2$).

Here we shall not take into account the higher-twist effects which are considered to be rather small as claimed in refs. [27].

In Fig.2 we have shown the QCD coupling constant α_s as a function of Γ. Here we note that α_s varies significantly with the change of the Γ. The uncertainty in Γ due to target mass effects gives rise to that for the QCD coupling constant $\alpha_s(Q^2 = 2.5\text{GeV}^2)$. Namely, for the variation

$$0.132 \leq \Gamma \leq 0.190 \quad \text{(Analysis I)}, \quad (11)$$

we obtain the ambiguity for α_s

$$0.213 \leq \alpha_s(Q^2 = 2.5\text{GeV}^2) \leq 0.474 \quad \text{(Analysis I)}. \quad (12)$$

For the analyses II and III, we have

$$0.134 \leq \Gamma \leq 0.188 \quad \text{(Analysis II)}, \qquad 0.148 \leq \Gamma \leq 0.174 \quad \text{(Analysis III)}, \quad (13)$$

which lead to the ambiguity for α_s

$$\begin{aligned} 0.228 &\leq \alpha_s(Q^2 = 2.5\text{GeV}^2) \leq 0.469 \quad \text{(Analysis II)}, \\ 0.315 &\leq \alpha_s(Q^2 = 2.5\text{GeV}^2) \leq 0.424 \quad \text{(Analysis III)}. \end{aligned} \quad (14)$$

To summarize, in this paper we have examined the possible corrections to the Bjorken sum rule coming from target mass effects. We have found that at relatively

small Q^2 where the QCD effect is significant, the target mass effects are also non-negligible. We found that to test the target mass correction precisely, we need accurate data for $A_2(x, Q^2)$. In determining the QCD coupling constant α_s from the Bjorken sum rule, there appears uncertainty due to target mass effects. This uncertainty can also be removed by the experimental data on $A_2(x, Q^2)$.

Although in this paper we have confined ourselves to the target mass effects in the Bjorken sum rule, the similar analysis can be carried out for the Ellis-Jaffe sum rule which will be discussed elsewhere [28].

We hope that future experiments at CERN, SLAC and DESY will provide us with data on A_1 possessing higher statistics as well as the data on A_2 with high accuracy which will enable us to study g_2 structure functions and also target mass effects more in detail.

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Figure Captions

Fig.1 The upper bound for the target mass correction $\Delta\Gamma$, $\Delta\Gamma_{u.b.}$, as a function of Q^2. The solid, short-dashed and long-dashed lines show the upper bounds for the analyses I, II and III, respectively.

Fig.2 The Bjorken sum rule Γ versus the QCD coupling constant α_s at $Q^2 = 2.5\mathrm{GeV}^2$. The dot-dashed line corresponds to the Ellis-Karliner's analysis. The solid, short-dashed and long-dashed lines show the upper and lower limits of Γ with the corrections for the analyses I, II and III, respectively.

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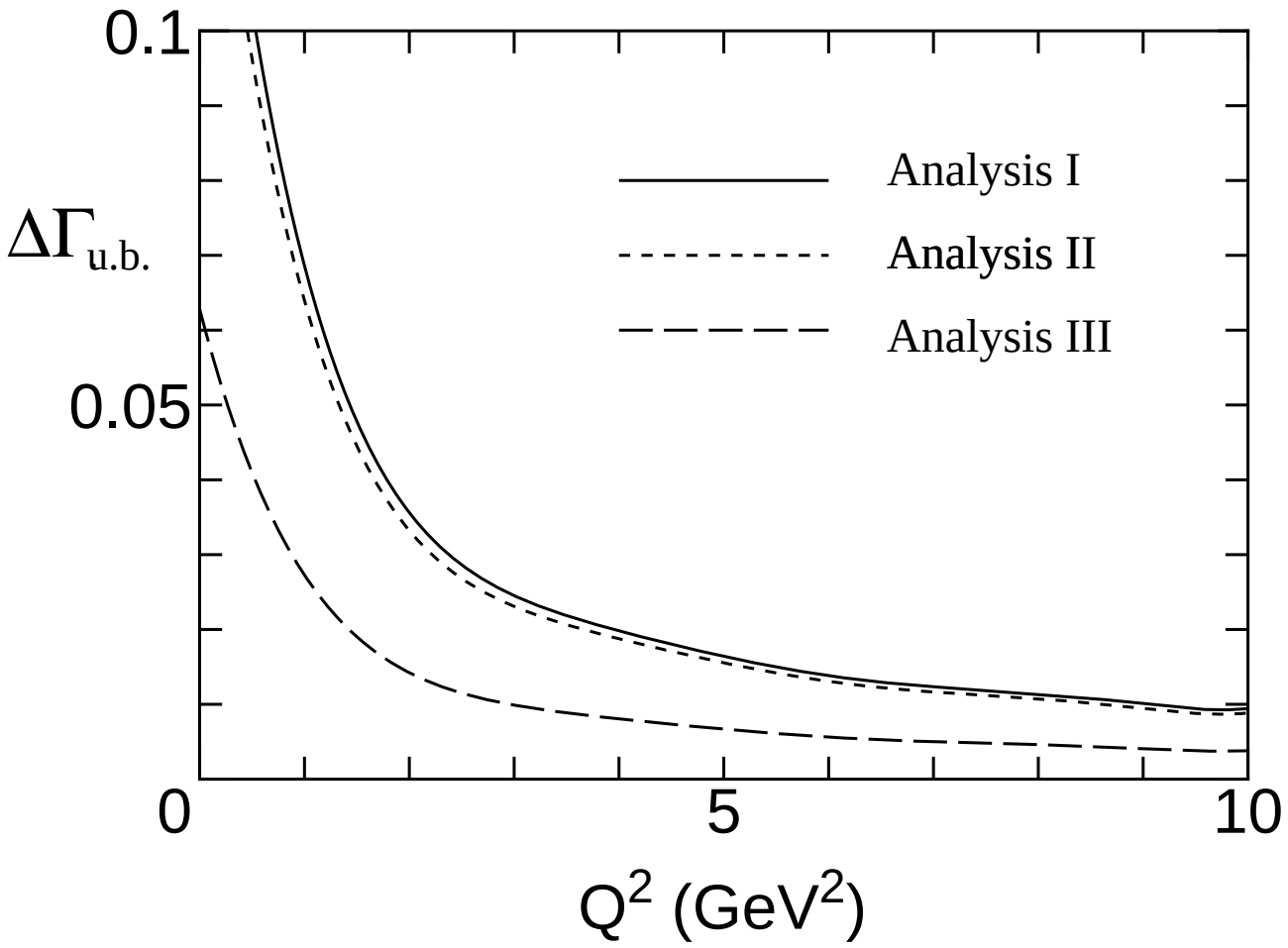
0.1
0.05
0
5
10
$\Delta\Gamma_{u.b.}$
Q^2 (GeV^2)
Analysis I
Analysis II
Analysis III

Fig.1

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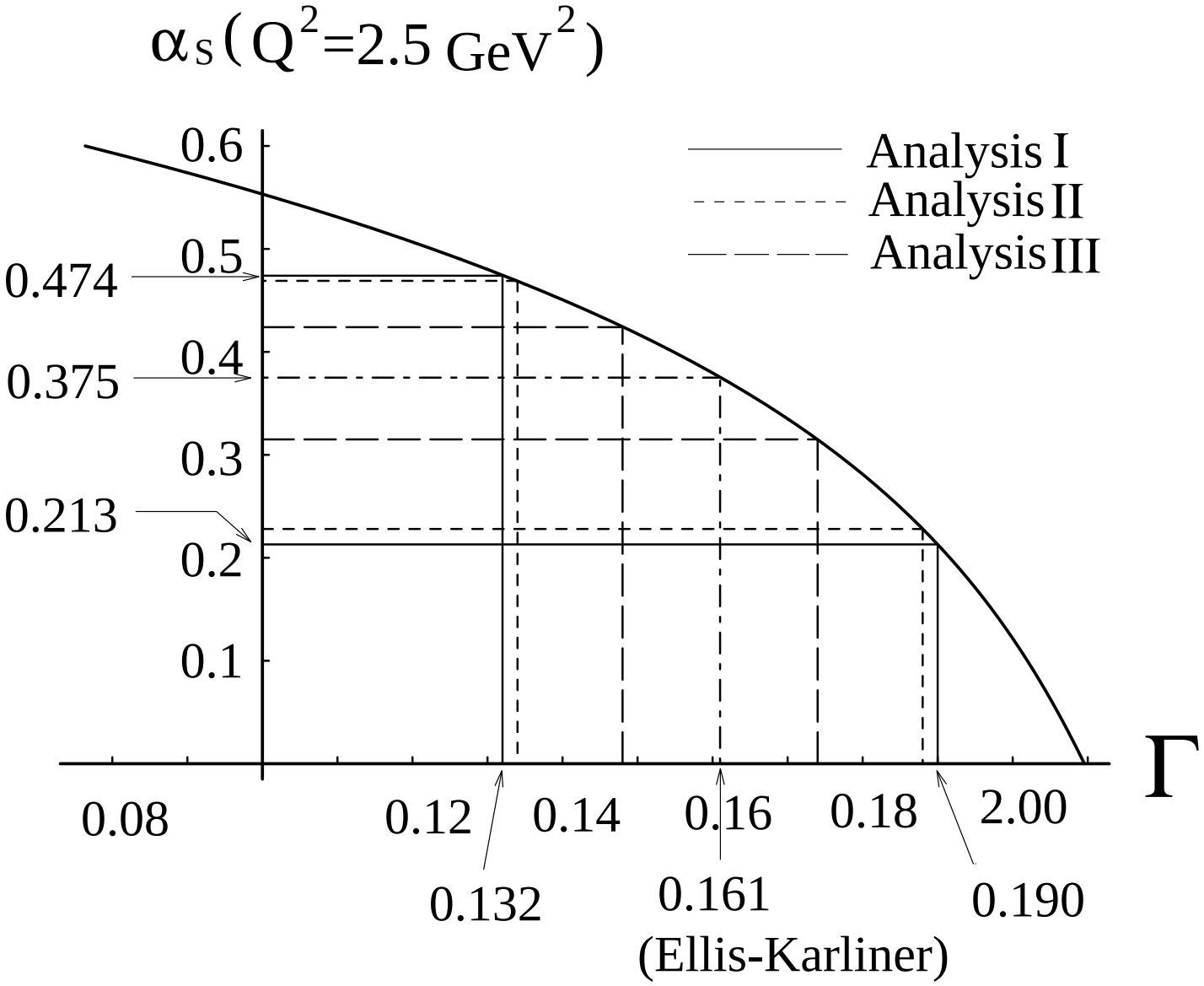
$\alpha_S(Q^2=2.5\,\mathrm{GeV}^2)$
0.6
0.5
0.474
0.4
0.375
0.3
0.213
0.2
0.1
Analysis I
Analysis II
Analysis III
Γ
0.08
0.12
0.14
0.16
0.18
2.00
0.132
0.161
(Ellis-Karliner)
0.190

Fig.2